

### Definitions

**Vector:** a quantity that has both direction and magnitude, or size, and is represented in the coordinate plane by an arrow drawn from one point to another.

**Component form:** a representation of a vector giving its x and y components; e.g.  $\langle 3, -4 \rangle$

**Transformation:** a function that moves or changes a figure in some way to produce a new figure.

**Preimage:** the original figure before the transformation.

**Image:** the figure after the transformation.

**Rigid motion:** any transformation that does not change the shape or size of the figure. Also, **congruence transformation**.

**Translation:** a transformation that moves every point of a figure the same distance in the same direction.

**Reflection:** a transformation that uses a line like a mirror to reflect a figure.

**Rotation:** a transformation in which every point of a figure is turned about a fixed point the same angle while keeping the same distance from that fixed point.

**Line of reflection:** the mirror line of a reflection.

**Glide reflection:** a transformation involving a translation followed by a reflection.

### Coordinate Rules for Transformations

Reflection across the x-axis:  $(a,b) \rightarrow (a,-b)$

Reflection across the y-axis:  $(a,b) \rightarrow (-a,b)$

Reflection across the line  $y = x$ :  $(a,b) \rightarrow (b,a)$

Reflection across the line  $y = -x$ :  $(a,b) \rightarrow (-b,-a)$

### Theorems and Postulates

**Translation Postulate:** A translation is a rigid motion (isometry).

**Reflection Postulate:** A reflection is a rigid motion.

**Rotation Postulate:** A rotation is a rigid motion.

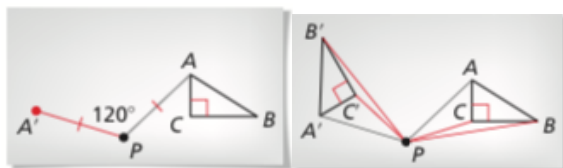
**Composition Theorem:** The composition of two (or more) rigid motions is a rigid motion.

**Reflections in Parallel Lines Theorem:** If lines  $k$  and  $m$  are parallel, then a reflection in line  $k$  followed by a reflection in line  $m$  is the same as a translation.

**Reflections in Intersecting Lines Theorem:** If lines  $k$  and  $m$  intersect at point  $P$ , then a reflection in line  $k$  followed by a reflection in line  $m$  is the same as a rotation about point  $P$ .

### Constructions

Draw a Rotation (P. 190)



**Line symmetry:** when a figure can be mapped onto itself by a reflection.

**Center of rotation:** the fixed point of reference used in a rotation.

**Angle of rotation:** the angle used by every point in a figure when rotating.

**Rotational symmetry:** when a figure can be mapped onto itself by a rotation of  $180^\circ$  or less about the center of the figure.

**Center of symmetry:** the center of rotation when a figure has rotational symmetry.

**Congruent figures:** two figures in which a rigid motion or composition of rigid motions maps one onto the other.

**Dilation:** a transformation in which a figure is enlarged or reduced with respect to a fixed point  $C$ .

**Scale factor:** the ratio of lengths of the corresponding sides of the image and preimage.

**Center of dilation:** the fixed point in a dilation.

**Similarity transformation:** one or more dilations and rigid motions.

**Similar figures:** two figures in which a similarity transformation maps one onto the other.

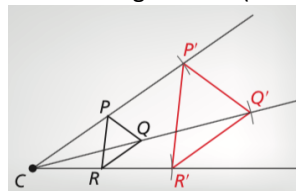
Rotation of  $90^\circ$  about the origin:  $(a,b) \rightarrow (-b,a)$

Rotation of  $180^\circ$  about the origin:  $(a,b) \rightarrow (-a,-b)$

Rotation of  $270^\circ$  ( $-90^\circ$ ) about the origin:  $(a,b) \rightarrow (b,-a)$

Dilation by scale factor  $k$  using  $C(0,0)$ :  $(a,b) \rightarrow (ka, kb)$

Constructing Dilation (P. 210)



Show all work!!!

1) Reflect the following figures with the given vertices across the given lines.

a) A(2, 3), B(-1, 5), C(4, -1);  $y = x$

b) U(-8, 2), V(-3, -1), W(3, 3); y-axis

c) E(-3, -2), F(6, -4), G(-2, 1); x-axis

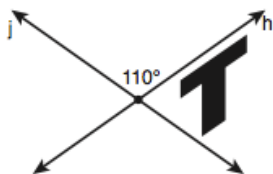
3) Rotate  $\triangle RST$  with vertices R(-1, 4), S(2, 1), and T(3, -3) about the origin by the given angle.

a)  $90^\circ$

b)  $180^\circ$

c)  $-90^\circ$

5) To create a logo for new sweatshirts, a designer reflects the letter T across line h. That image is then reflected across line j. Describe a single transformation that moves the figure from its starting position to its final position. Also, draw two lines of reflection that produce an equivalent transformation.



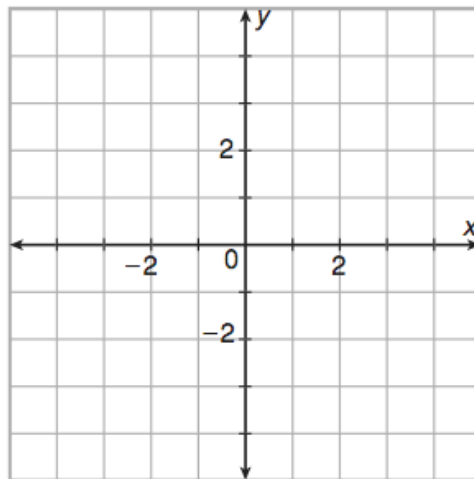
2) Translate the figure with the given vertices along the given vector.

a) G(8, 2), H(-4, 5), I(3, -1);  $\langle -2, 0 \rangle$

b) S(0, -7), T(-4, 4), U(-5, 2), V(8, 1);  $\langle -4, 5 \rangle$

c) D(-3, -1), E(5, -3), F(-2, 2);  $\langle 3, -1 \rangle$

4) P(5, -2), Q(1, -4), and R(-3, 3). Translate  $\triangle PQR$  along the vector  $\langle -2, 1 \rangle$  and then reflect it across the x-axis.



6) Rectangle WXYZ has vertices W(-3, -1), X(-3, 3), Y(5, 3), and Z(5, -1).

a. Find the perimeter and area of the rectangle.

b. Dilate the rectangle using a scale factor of 3. Find the perimeter and area of the dilated rectangle. Compare with the original rectangle. What do you notice?

c. Repeat part (b) using a scale factor of  $1/4$ .

d. Make a conjecture for how the perimeter and area change when a figure is dilated.